hw set 20
sect 8.1
p565

3. List 1st 6 terms of $a_n = \frac{n}{2n+1}$. Find the Limit if there is one.

$a_1 = \frac{1}{3}$, $a_2 = \frac{2}{5}$, $a_3 = \frac{3}{7}$, $a_4 = \frac{4}{9}$, $a_5 = \frac{5}{11}$, $a_6 = \frac{6}{13}$

$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \left(\frac{n}{2n+1}\right) = \frac{1}{2}$

6. Find $a_n$. $\{-\frac{1}{4}, \frac{2}{9}, -\frac{3}{16}, \frac{4}{25}, \ldots\}$

$a_n = (-1)^n \frac{n}{(n+1)^2}$

7. Find $a_n$. $\{2, 7, 12, 17 \ldots\}$

$a_n = 2 + 5(n-1) = 5n - 3$

9. $a_n = \frac{3+5n^2}{n+n^2}$

$\lim_{n\to\infty} \left(\frac{3+5n^2}{n+n^2}\right) = 5$

11. $a_n = \frac{2^n}{3^{n+1}}$

$\lim_{n\to\infty} \left(\frac{2^n}{3^{n+1}}\right) = \frac{1}{3} \lim_{n\to\infty} \left(\frac{2}{3}\right)^n = \frac{1}{3}\cdot 0 = 0$

12. $a_n = \frac{\sqrt{n}}{1+\sqrt{n}}$

$\lim_{n\to\infty} \left(\frac{\sqrt{n}}{1+\sqrt{n}}\right) = 1$

13. $a_n = \frac{(n+2)!}{n!}$

$\lim_{n\to\infty} \frac{(n+2)!}{n!} = \lim_{n\to\infty} (n+2)(n+1) = \infty$

$\therefore$ diverges

23. $a_n = \left(1 + \frac{2}{n}\right)^n$

$\lim_{n\to\infty} \left(1+\frac{2}{n}\right)^n \Rightarrow \lim_{n\to\infty} \ln\left(1+2/n\right)^n = \lim_{n\to\infty} \frac{\ln(1+2/n)}{1/n} = \lim_{n\to\infty} \frac{\frac{-2/n^2}{(1+2/n)}}{-1/n^2} = \lim_{n\to\infty} \frac{2}{1+2/n} = 2$

so $\lim_{n\to\infty} (1+2/n)^n = e^2$

41. $a_n = \frac{1}{2n+3}$

decreasing : $a_{n-1} = \frac{1}{2n-1} > \frac{1}{2n+3} = a_n$

$\forall n$, $1/3 \geq a_n \geq 0$ (bounded)

40. Find Limit $\{\sqrt{2}, \sqrt{2\sqrt{2}}, \sqrt{2\sqrt{2\sqrt{2}}}, \ldots\}$

$a_n = \sqrt{2a_{n-1}}$

suppose limit exists. then

$a_n \to a$ & $a_{n-1} \to a$

we get

$a = \sqrt{2a}$ (from above)

$a^2 = 2a$

$a = 2$ = Limit

44. $a_n = n + \frac{1}{n}$ $n \geq 1$

increasing: $a_{n-1} = (n-1) + \frac{1}{n-1} \leq n + \frac{1}{n} = a_n$

$\forall n$ $2 \leq a_n$ (only lower bound)

52. $a_1 = 1$, $a_{n+1} = 1 + \frac{1}{1+a_n}$

$a_1 = 1$, $a_2 = 1.5 = 3/2$

$a_3 = 1.4$, $a_4 = 1.41\overline{6}$

$a_5 = 41/29$, $a_6 = 99/70$

$a_7 = \frac{239}{169}$, $a_8 = \frac{577}{408} = 1.414216$

notice $a_n \to \sqrt{2}$